The Soul at the Razor's Edge

L'Âme au bout d'un rasoir

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Intervening between logic and biology, a statistician who, first trained in mathematics, went then towards linguistics and other human sciences, should be cautious. But if he refrains both from overly technical formulas and overly vague sentences what will remain for him to say?
I will apply myself therefore to raise, by the interplay of formulas, sufficiently suggestive images to illustrate, animate and circumscribe a number of old maxims. Even if the authority of erudition is no longer today met with as formerly among the scholarly in long dress of Bologna or of Baghdad, I will go in quest of, sometimes far from us, some learned testimonies to the universality of my subject.

§1 Operations of the Mind

On the threshold of his commentary on the {De Interpretatione} of Aristotle, Saint Thomas recalls that, according to the philosopher, the operation of the mind is double: intelligence of the indivisibles (of that which every thing is in itself); then formation of affirmative or negative propositions. To that a third operation is added, reasoning, through which reason advances from the known to the unknown.
But is this not to run too fast?
Turning around the edifice, impeccable in itself, of the dialectic, Francis Bacon is ironical (*Novum Organum*, “new tool”, aphorism XIV):
“The syllogism is made of propositions, the propositions of words, and the words are the tokens of notions. Therefore if, at the base, the notions themselves are confused, abstracted rashly from things, what one constructs above is devoid of solidity“. And to conclude:

Itaque spes est una in inductione vera.
“There is no hope except in a true induction. “

What is the illustrious Chancellor dreaming of?
By a true induction, one would define terms which would not be vain bags of wind but like networks judiciously placed there where reality divides off from itself, and inserting in their mesh an abundance of things: the interplay of words would then surely mimic the interplay of these things. Thus, counter to what is too frequent today, abstraction must not be understood at all as the state of that which is autonomous (regulating itself according to its own laws, before dictating to science), but as a process that without ever withdrawing from reality, takes its forms from it.

Assuredly if Bacon were to take up again amongst us the pen, quickly wearying of scratching the invertebrate discourses of many an idealist author who glorifies in perusing rhyme and reason, he would find in mathematics and its logic, which are the new dialectic, worthy adversaries.

No tool, however - even if it would be this *novius organum,* "newer tool", so powerful for induction, that is the electronic computer confronting a profusion of facts through statistical calculation - would give to man a total direct hold on reality, even if only practical.

For want of being able to encompass, one takes with tweezers: that suffices for the mysterious light of intelligence to touch being.
But without claiming, even were this planned, to make a clear sweep (and how could one being placed as we are in the midst of things? never holding beginning nor end; nor the infinitely small, nor the infinitely large; nor the part nor the whole; nor our own thought no more than that of the world), without ceasing to name, affirm, reason, as Aristotle saw it, one will allow us to turn back to scrutinize definitions. All of biology, is it not in the single word of life?

§2 Definition and Reasoning in Classical Logic

Let us recall, according to an excellent treatise on philosophy {Régis Jolivet, Traité de Philosophie (Treatise of Philosophy); Vol.1, Section 46, 2nd edn., Vitte Lyon-Paris, 1945.}, what one means classically in logic by extension and comprehension: this will be the occasion then to contrast the thought of the biologist with contemporary formalist trends.

"One can", writes R. Jolivet, "consider an idea and subsequently a term from the point of view of comprehension and from the point of view of extension ...".

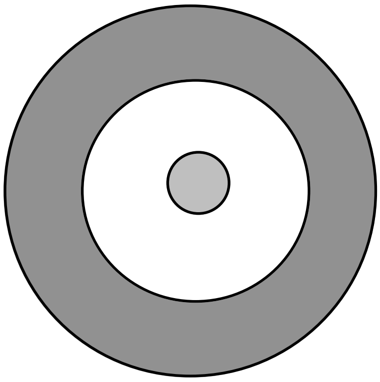
1 Comprehension is the content of an idea, i.e. { the set of elements of which an idea is composed}. Thus comprehension of the idea of man implies the following elements: being, living, endowed with senses, and with reason.

2 Extension is the set of the subjects which the idea suits.
It is thus that the concept of "animal" of which the extension is larger (e.g.) than that of man is called superior concept:
the concepts which enter its extension are its inferiors or its {subjective parts}.

The same author {*Ibid*. Section 36.} shows well how, from the point of view of the extension, one obtains a sort of visual or geometric representation of the syllogism. It is thus that the schema of Euler symbolizes the extension of terms by three concentric circles, sketched on the next page. Here we consider two ideas (or concepts):

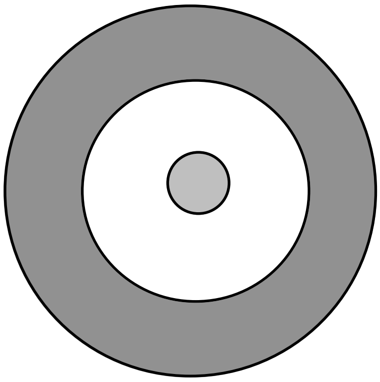
D (mortal: subject to Death) ; M (Man)

and one term, or individual: t (Peter)



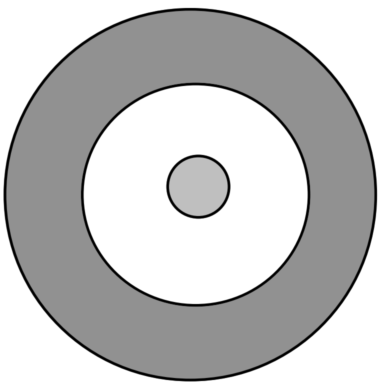
M D
Every man is mortal.
[D outside (dark grey)]

{*Man*} (M) is completely included
in the extension of {*Mortal*} (D).
[M middle (white)]



t M
Now, Peter is a Man.
[t center (light grey)]

Peter (t) is
in the extension of {*Man*} (M).
[M middle (white)]



t D
therefore, Peter is a Mortal.
[t center (light grey)]

Peter (t) is
in the extension of {*Mortal*} (D).
[D outside (dark grey)]

In this *visual or geometric representation*, the mathematician of today recognizes *sets*, and he is filled with wonder that in a time when set theory fever did not yet burn the youngest foreheads, the great name of Euler found a place in a school manual of logic.
But further critiques, by Jolivet, came about:
“The *disadvantages* of the exclusive usage of this procedure are:
first to *systematically materialize an essentially intellectual operation*, lying on a *vision of the mind* and irreducible to any combination of figures, - then to *get the mind used to a sort of “logic of classes”* which, abandoning the objective content of ideas, i.e. of objects of thought, does not consider them more than as empty and purely arbitrary frames, susceptible to be replaced by some sign or other. Instead of considering the type of logical necessity that links the conclusion to the premises and that constitutes the essence of the reasoning, extensivist logic ends by not considering anything else than the exterior sign which is supposed to symbolize logical necessity, but which, in fact, conceals it from the mind."

All things considered, we subscribe to these critiques, … in as much as we understand them. What does *logical necessity* mean? We appreciate enough the depth of thought of R. Jolivet to believe that, with him, these words are more loaded with meaning than *virtus dormitiva* Molière's doctor. But, just as an empty box resembles to the sight a full box, likewise the way of speaking of philosophers often leaves a mathematician in doubt: What has he said? Did he say anything? Moreover if a *vision of the mind* is irreducible to any combination of figures it is left that one cannot think without a figure:

νοεῖν οὐκ ἔστιν ἄνευ φαντάσματος
noein ouk estin aneu phantasmatos
without an image (present in the mind) it is impossible to think

writes Aristotle in:

περὶ μνήμης καὶ ἀναμνήσεως, peri mnêmês kai anamnêseôs
on Memory (impression left) and Reminiscence (to make actually present).

The object of the vision of the mind is not a figure but it is suggested by the concept of many images (as the concept of triangle by many drawn triangles).
Now mathematical language depicts with exactitude and conciseness; we will make use of it to suggest the meaning of the critiques formulated by R. Jolivet.

§3 Logic and Formulas

Let us leave contemporary mathematical logic, where notions are very precise if not perfectly appropriate for the thought of him who scrutinizes nature. Commonly, reasoning expresses itself in a language: before asking oneself if one is telling the truth one can enquire as to whether one has just spoken sentences: likewise mathematical logic presupposes a formal grammar.
Fundamentally, this grammar is only a system of rules to decide, among finite sequences of symbols (taken in a set or alphabet decreed at the start), which are well-formed expressions; and, in particular, propositions (or terms, or sentences). Logic then considers these propositions as being able to receive a value; and it provides these rules, called deduction rules or schemas, to attribute a value to certain propositions, the value of certain others being known. The reader, supposedly guessing what we are talking about, will get irritated perhaps that we have not been saying: rules permitting to affirm that certain propositions are true or false, a certain number of others being supposed true.

This is because we are not yet where we want to be: the set of possible values for a proposition is not necessarily the set of two elements {*true*, *false*}; this can be a set of more than two elements, to which, in the formal interplay that is ours here, one does not give a name. Would one name them, one would not have rejected the principle of the excluded middle according to which it is not given to a proposition to be other than true or false (a third outcome is excluded, *tertium non datur*); because what we call here well-formed propositions are just assemblages of signs to which it remains to give a meaning, a real referent; which has not yet been done and which, once the correspondence rules decreed, is not perhaps feasible for every sentence (hence an example of the third outcome, for which the philosopher would be wrong to blame the logician).

The correspondence between terms of a formal system and truths of a natural domain (e.g. *biology*) has hardly at all been scrutinized, for its own sake - going even into the detail. Around 1940 there was current an ambitious project, under more than one great name, among them that of R. Carnap, for an

International Encyclopedia of Unified Science.

The ultimate objective was without doubt to start from elementary truths of experience tabulated following the rules of a rigorous positivism, to rise up by computational logic as far as the general terms of the sciences; but only methodological volumes saw the light of day. In practice the logical description of the real passes through the mathematical description of the latter.

The formalists do not appear to us to have had, on the appropriateness of mathematics to the real, fertile or profound views.

{One should however credit the formalists with their contribution to the conception of electronic computers; from which poceeds an indirect, but in our view essential, contribution to the mathematical elaboration of real data; we will return to this point.}

On the other hand, they made spectacular discoveries on what can be the appropriateness of mathematics of a logical system of signs. Here is not the place to try only to present the works of Kurt Gœdel or of Paul Cohen; but to conceive of what becomes in contemporary mathematical logic *extension* and *comprehension*, relations must be considered between the statements of a formal system (or language of the logic) on the one hand, and, on the other hand, the mathematical objects relative to which these statements take on meaning; this is what we now intend to do briefly.

§4 Syntax of Symbolic Logic

Up to now we have not said anything about the language where logical propositions are written. We wanted thus to emphasize that, for the logician, this language is firstly an articulated interplay of signs, of which the rules can be freely chosen, and not a simplified and perfectly regular translation of one's mother tongue. Having posed this principle, it remains that the variations of language commonly met with, of one author to another, and even with a single author, of one work to another, leave intact a common background of structure which is not incomprehensible for an honest man practicing natural languages.

So much so that the very simple language that we present below offers to a true logician everything necessary to pose the most profound problems.
The alphabet consists of the following 12 symbols:

z ; **&** ; **V** ;
x ; **c** ; **R** ; **=** ;
∀ ;
(; **)** ; **ʻ** ; **,** ;

Rather than set out a formal grammar, let us suggest through correspondence with common languages the rules of formation of propositions.

z is an adverb (or unary conjunction) which, placed before a proposition, changes it into its negation.

& and **V** are the binary conjunctions ("and" and "or") that placed between two propositions make of them a new proposition.

c (possibly assigned any number of raised accents: **cʻ**, **cʻʻ**, **cʻʻʻ**, designates a constant individual.

x (possibly assigned raised accents), designates an individual variable: a variable is like an indefinite pronoun, all occurrences of which in the sentence would be linked by a thread, similar to that which links the relative to its antecedent: one knows thus that it is always a matter of the same indeterminate individual. Natural languages are far from having this unlimited clarity!

R is a relational constant or predicate: if **R** is assigned n lower indices one has an n-ary predicate (the number of raised indices is only a convenience that ensures an infinite vocabulary): for example

R′,,,(x,c,x')

is a proposition that expresses the relation **R′,,,** between three individuals, **x,c,x'**, of which two are indeterminate.

= is a particular binary relation, equality.

∀ is the universal quantifier: *for all* ; placed at the head of a complex proposition (formed of predicates, conjunctions, variable or constant individuals), it expresses the possibility that the relation that expresses the proposition exists, irrespective of the particular individual taken for **x**.

Finally, on the way, we have noticed the use of parentheses and of accents.

Once the language has been described, it remains to give the principles or schemas of deduction permitting to pass from a set of propositions or premises supposed to have the value *true* to a conclusion of the same value. From these schemas, which are the logic as such, we will limit ourselves to say that they translate into formulas the, all in all, natural and classical rational principles: as for deduction, contemporary formalism is not against the Aristotelian syllogism (set out in the *Prior Analytics*) any more than modern algebra is not against the arithmetic calculations of Diophantus. In our view, the neo-scholastic philosophers, who are indignant at seeing the syllogism abandoned, extend without due cause to the entirety of mathematical logic a hostility that they should reserve just for nominalism (professed it is true by a number of mathematicians; who, we were recalling above, understand abstraction as a state, not as a process).

§5 Relations and Models

Syntax and logic being fixed, one can constitute a particular theory as within the framework of a natural language; one reasons from basic notions and truths. The language described in the previous section contains two signs **c** and **R,** that, through the adding of accents, will furnish the theory with the vocabulary that one desires.

Sometimes one introduces infinite sets of individuals and of predicates; but in fixing a finite number of them one has already matter for the deepest research. With the vocabulary thereby decreed basic postulates of the theory are written, which are the propositions from the truth of which one proposes to deduce the logical consequences.

One must be careful even if the quantifier *for all* and the values (x, x', …) refer, in their meaning, to the set of all possible individuals, this set (for which the particular theory has only fixed some constant individuals: { $\mathbf{c}$, $\mathbf{c'}$, $\mathbf{c''}$,...}) stays in the dark.
Logical computation itself operates without naming it and without even postulating it. If we have spoken about it above, it is to suggest the finality, the possible utility of the interplay of formulas. To introduce such a set that we will name: $\mathbf{I}$, is to give a *model* to the theory.

And it is not sufficient that $\mathbf{I}$ includes all the constants: { $\mathbf{c}$, $\mathbf{c'}$, $\mathbf{c''}$,...}, used in the basic vocabulary; truth relations must still be specified for each predicate of this vocabulary: for example, if { $\mathbf{i}$, $\mathbf{i'}$, $\mathbf{i''}$,...}, are particular individuals of the set $\mathbf{I}$, one should say if : $\mathbf{R'',,,(\,i,\,i',\,i'')}$, is true or not;
finally the relations thus specified must, and this is essential, conform to the premises of the theory (for example, if one has as premises:

$$\forall \mathbf{x},\ \forall \mathbf{x'} \quad \mathbf{R'',,,(\,x,\,x',\,x'')}\ ,$$

one must have in particular : $\mathbf{R'',,,(\,i,\,i',\,i'')}$.

Mathematical logic demonstrates, about the existence of a model and the non-contradiction of a theory, a number of theorems (due, principally, to Kurt Gœdel), which are of real philosophical impact.

On the one hand, briefly, a model exists if and only if the theory is non-contradictory (if one cannot demonstrate simultaneously there a proposition and its negation).
On the other hand, to establish that an even remotely rich theory (e.g. the arithmetic of whole numbers) is non-contradictory requires the help of an even richer theory (for which one can claim even less that it is non-contradictory).
Certain mathematicians conclude that hypothetical mathematical beings must be the least possible postulated and they give precise rules to their asceticism. This theory, which is that of intuitionism, has very resolute adversaries.

Without having anything profound to contribute to this internal debate in mathematics we will suggest that the coherence of a theory is best demonstrated by its appropriateness for natural phenomena of a certain order: for, if the appropriateness is perfect, nature supplies a model to the theory; and if, as it is likely, the appropriateness is only imperfect, one is at least assured that the theory will be able, without disappearing, to improve itself and eliminate its contradictions, becoming a better approximation of the real.

To demonstrate, within a theory, that a proposition and its negation are true can appear an absolute vice; but this demonstration can result from a long chain, that a distinction introduced into an axiom suffices to break.

One may refer, here, to what has been said, in “Choriogenèse”, about Henri Cartan’s “théorie des Filtres” as applied to the concept of truth.

§6 Notions in Formulæ

With a view to this confrontation with the real, let us see, from examples, what is in mathematical logic a notion, its extension, then its comprehension.

First let $\mathbf{R}$ be a unary predicate: the extension of $\mathbf{R}$ is fixed with the model: it is the set of individuals, **i**, for which is specified that $\mathbf{R,}(\mathbf{i})$ is true. But there are in the theory more complex notions than the basic unary predicates; for example the statement:

$$\forall \mathbf{x} \;\; \mathbf{R,,}(\mathbf{i}, \mathbf{x}) \;\; \mathrm{V} \;\; \mathbf{R'',,,}(\mathbf{i}, \mathbf{x}, \mathbf{x})$$

(whatever the determined $\mathbf{x}$ either $\mathbf{R,,}(\mathbf{i}, \mathbf{x})$ or $\mathbf{R'',,,}(\mathbf{i}, \mathbf{x}, \mathbf{x})$ is true) defines for $\mathbf{i}$ a notion $\mathbf{N}$: the statement is true for fhe individuals, $\mathbf{i}$, of a certain subset of $\mathbf{I}$, that is, by definition, the extension of $\mathbf{N}$ and is commonly denoted:

$$\mathbf{I}\,(\mathbf{N}) = \{\, \mathbf{i} \mid \mathbf{i} \in \mathbf{I} \;;\; \forall \mathbf{x} \;\; \mathbf{R,,}(\mathbf{i}, \mathbf{x}) \;\; \mathrm{V} \;\; \mathbf{R'',,,}(\mathbf{i}, \mathbf{x}, \mathbf{x}) \,\} .$$

With every similar statement that includes, beyond predicates and a variable $\mathbf{x}$ (there could be several of them) linked to the quantifier $\forall$, the symbol $\mathbf{i}$ of a free individual (not linked to a quantifier) is associated a notion.
Among the parts (subsets) of $\mathbf{I}$, certain only can, like $\mathbf{I}\,(\mathbf{N})$, be described as the extension of a notion defined within the theory of which $\mathbf{I}$ is a model.

As for comprehension, let us consider first a notion $\mathbf{N'}$ defined by the conjunction of three unary predicates, $\mathbf{R,}$, $\mathbf{R',}$ and $\mathbf{R'',}$, and for which the extension is:

$$\mathbf{I}\,(\mathbf{N'}) = \{\, \mathbf{i} \mid \mathbf{i} \in \mathbf{I} \;;\; \mathbf{R,}(\mathbf{i}) \wedge \mathbf{R',}(\mathbf{i}) \wedge \mathbf{R'',}(\mathbf{i}) \,\} \;;$$

(i.e. the set of individuals for which $\mathbf{R,}(\mathbf{i})$, $\mathbf{R',}(\mathbf{i})$ and $\mathbf{R'',}(\mathbf{i})$ are simultaneously true). It is natural to say that the comprehension of this notion is the set of three predicates $\{\, \mathbf{R,}$, $\mathbf{R',}$ and $\mathbf{R'',} \,\}$. But in proceeding thus it is impossible to define every notion by its comprehension. It is more correct to say that on the one hand the notions, because they are defined by means of basic predicates (or properties), are in a certain sense defined as comprehension rather than as extension; and on the other hand that the relations between notions (e.g. that $\mathbf{N'}$ is a consequence of $\mathbf{N}$) can be expressed not only in extension, in the context of a model:

$\mathbf{I}(\mathbf{N'}) \supset \mathbf{I}(\mathbf{N})$: every $\mathbf{i}$ that possesses $\mathbf{N}$ possesses $\mathbf{N'}$;
but also independently of any model, as a proposition of the theory (which will be for $\mathbf{N}$ and $\mathbf{N'}$):

$$\forall \mathbf{x'} \,(\forall \mathbf{x} \,:\, \mathbf{R'',,,}(\mathbf{x'}, \mathbf{x}, \mathbf{x})) \;\; \mathrm{V} \;\; \mathbf{z}\,(\, \mathbf{R,}(\mathbf{x'}) \wedge \mathbf{R',}(\mathbf{x'}) \wedge \mathbf{R'',}(\mathbf{x'})) ;$$

that is, that every $\mathbf{x'}$ either possesses $\mathbf{N'}$ or does not possess $\mathbf{N}$.

If one accepts this point of view, formal logic works far more relying on comprehension than on extension; and the philosophical critiques of authors such as R. Jolivet have nothing to say against that. But one must be careful in that being opposed to a *logic of classes* founded on extension, these philosophers wish to prevent the spirit to stop at symbols while losing sight of the *objective content of ideas*, the content, which alone, according to them, founds *logical necessity*.
By *objective content*, it must be understood these domains of real facts, which the notions do not lock up perfectly enough for the articulations to be able to be enumerated in axioms;

logical necessity results finally always from a to and fro between things and words: the spirit seizes things by words and calculates on the latter; but it has to test if the separation between things and words (linked by a more or less narrow, but non-identified, analogy) does not render illusory to affirm from the former the conclusions obtained on the latter. On natural relations between facts and concepts we shall speak now.

§7 Inductive Definition

At the threshold of an Introduction to biology destined for university students (M. Durand and P. Favard: "La Cellule" -The Cell-, Hermann, Paris; 1967), our colleague Professor P. Favard writes:
"Rather than making abstract reconstruction of an ideal cell, we will describe the organisation of cells belonging to very dissimilar organisms… For in their infinite diversity of organisation, the cells are only, all things considered, variations on a single theme…".
And to propose a series of examples; whether of cells incorporated in a pluricellular being, animal or vegetal; or of unicellular organisms -- protozoa, bacteria or green algae.
Thus for P. Favard, the idea of cell is not a model, a type that the abstraction would be able to fix; it is that which several particular living entities have as analogs.
The idea is not conceived if the spirit does not move from the particular to the general; to this inductive movement the author prompts the reader.

Not only the abstraction conceived as the state of that which is separated from the real is vain, but more the result of the process of abstraction is elusive: only the process itself holds.
When, as is the case here for the cell, a notion is defined as a polygonal zone underpinned by some individuals, we will speak of an *inductive definition*. We will see with an example that multidimensional statistical analysis can offer the inductive definition a rigorous foundation in giving to our image of the polygonal zone a concrete realization in a plane or a space.

Let us propose now with S. Blaise to study the systematics of the genus *Myosotis* {cf. S. Blaise, J.-P. Briane and M.-O. Lebeaux, Le genre Myosotis;
in: J.-P. Benzécri *et al*., L'Analyse des Données, Vol. I, part C, n°. 1, Dunod, 1973.}.

"This genus, of great morphological homogeneity, includes a high enough number of species and of sub-species all the more difficult to discriminate, on the one hand, since there exist hybrids (intermediary forms) and since; on the other hand, the same vegetal individual with determined genetic capital can, in accordance with the conditions of its growth, develop strongly diverse forms. … The determination of a base of *Myosotis* rests on the descriptive characters which we will arrange in three classes:

A) *Apparent qualitative characters*: such are the color and the form of the corolla, the pilosity of the leaves, etc.

B) *Architectural qualitative characters, easily quantified*: we will cite the distance between two nodes, the number of leaves on such a branch …

C) *Microscopic characters*: these are palynological characters (*palyno* means referring to *pollen*); and karyological (number of chromosomes …) …

The set of these characters allows very sure determinations… But in daily botanical practice, the microscopic characters C) are not known; the floral characters A) cited in ancient flora are often unusable in the state considered of development and of conservation of the base. Hence the interest of a statistical study of the architectural macroscopic characters B), the most general and the most easily determined."

Let us sketch this study. Suppose that one considers 13 macroscopic characters;
with every base of *Myosotis* is thus associated a system of 13 numbers, that is, in geometric language, a point of the space R^{13}, with 13 dimensions; and with the set of *Myosotis* of every species that one has measured, there corresponds a cloud of points of R^{13}. In this space however the eye of man has seen nothing! But statistical computation, for which the computer is the indispensable tool, provides generally for a cloud of R^{13}, a satisfactory map in the plane, or, somtimes, in the usual three-dimensional space; (we say "generally" because the cloud could be too complex, and the density of the set of possible forms might not be expressed in 2 or 3 dimensions; this is not the case here, for *Myosotis*).

On the planar map one notices first that the two large groups of *Myosotis* that one distinguishes by form of pollen (or by color of the flowers) are not quite exactly superimposed. They have the same macroscopic architecture; more exactly the same architectural diversity; but within each of these groups the principal subdivisions have their own areas; with, in an intermediate position, the hybrids.

It is not the place here to set out even summarily the geometric principles of multidimensional statistical analysis. Let us say again only that from, a set, a cloud, a nebula, of individuals, of a natural or other order, described by a set of their properties (and not only by measurements: we may study bacterial clones described by traits of their metabolism…) computation associates coordinates, called *Factors*, {F1, F2, F3…} that can be considered two by two, on plane maps, (F1 x F2, F2 x F3) where gradations and qualitative oppositions, discontinuities that subdivide the field studied, are inscribed; the discontinuities being eventually made more precise using algorithms of classification.

The computer is indeed the *novius organum,* "newer tool", of inductive definition.

§8 Statistical Existence

Statistics is not a means of knowledge, draining into an idea the flood of facts: it is a mode of being. Collective entities are not only constructions, even if dynamic, of our spirit; they are defined in nature for an equilibrium, an exchange of information between individuals. Here is the definition that the eminent paleontologist, G.G. Simpson proposes for "species" (G.G. Simpson, Principles of Animal Taxonomy, Columbia University Press,1961, p. 65):

"That category cannot be naturally defined in terms of static pattern or morphology but only in terms of dynamic, evolutionary, genetical, concepts and relationships among and between populations… Taxonomic studies are always statistical in character. The true object of enquiry, the population in nature, can rarely be observed directly and entire…"

Revealed by statistical study of a sample (cf. *Myosotis* above) a species is in itself a sample that never exhausts the set of diverse possible forms; if the species is particularly few in number, or geographically broken up, it is possible that from heredity or mutations the equilibrium of these possible forms is unstable.

The statistical definition of species as a system of masses (the individuals) partially occupying a potential field (which is in the set, or more so in the space, of possible living things) conflicts with the definition by a finite and closed system of properties, for which mathematical logic studies conjunction and disjunction, writing formulas such as those presented above.

Simpson, whom the universe of the logical positivists does not attract, deplores that {*Ibid*., p. 24.} :
"Some present-day taxonomists advocate what is essentially a return to scholastic principles."

The term scholastic is ambiguous: for some, it serves to profess their fidelity to the Doctors of whom the most illustrious is Saint Thomas Aquinas; with the pen of others, this is a sign of contempt striking out what is however the so diverse and often contradictory totality of the philosophical production of an ignored epoch. Simpson, as for himself, is not well-versed in medieval thought. He knows by hearsay that Bacon and Descartes denounced a very perfectioned dialectic, but all the less capable of encompassing a real object. He notes that the abuse of the formalism today places science in similar peril. We can, without lacking respect to our masters of the 13th century, subscribe to these criticisms.

Of the statistical mode of being, vegetal ecology offers a remarkable example; here again let us cite a classic: M. Guinochet, { Logique et Dynamique du Peuplement Végétal (Logic and Dynamic of Vegetal Populating), Masson, Paris (1955), p. 63.} who writes:

"... Let us define in nature, for example on a lawn, a surface area of 1 m^2 (taken randomly), and let us note all species that one observes there. Let us enlarge this surface area to 2 m^2. Our list lengthens by some species and so on. Let us draw a plot, on which we have as abscissa the surface areas, and as ordinate the corresponding numbers of species. We obtain a curve that, after a more or less rapid climb becomes almost parallel to the abscissa axis.
Then if we continue to enlarge the surface area studied the curve will again become more or less rapidly climbing. The surface area corresponding to the first inflexion of the curve [we would rather say: at the start of the leveling off, JPB] is the minimum area necessary for one to have a chance of finding essentially all species of the "individual of association" studied; on the contrary the surface area corresponding to the second inflexion of the curve [end of the leveling off] is the maximum area that one can not go past without noting the species foreign to the individual of association considered ...". To what natural reality does the ' *individual of association*' correspond? The '*floristically homogeneous*' surface area for which M. Guinochet has just offered us an evidently very schematized definition. At a physico-chemical equilibrium, indeed microclimatic where all species participate, that on the basis offered by the minerals work out themselves the ground from which they live, in symbiosis with micro-organisms.

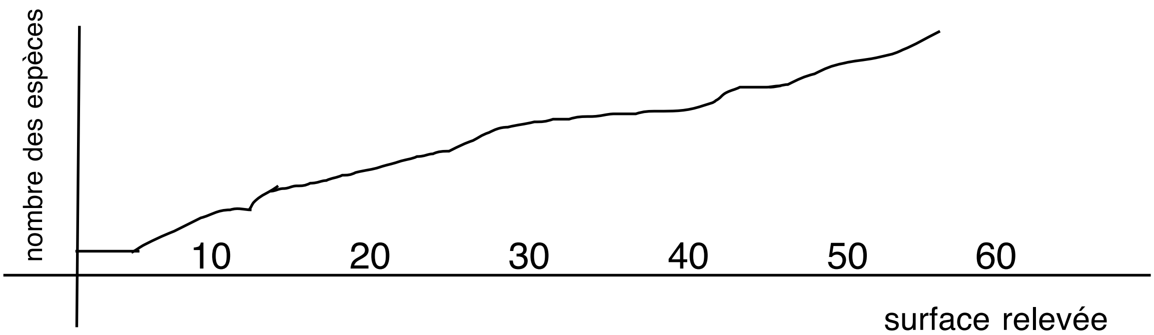


figure 1 : aire floristiquement homogène entre 15 et 40 m^2

However the ‘*individual*’ is not clearly circumscribed in the species (a surface area of 15 to 40 m^2 would be suitable as sketched in *figure 1*; where the number of species may be between 0 and 30); and it is also no longer logically defined by a set of characters -- the species that ought to be necessarily present there: a characteristic species can be missing from the table, a foreigner one can be fortuitously inserted there.
Notwithstanding, from these ‘*individuals*’, already materially in themselves difficult to get hold of, whose mode of being, hence *a fortiori* definition, subject to fluctuations are statistical, the ecologist constructs a taxonomy: in the same was as the botanist says that such a vegetal individual pertains to such a species, the ecologist recognizes in diverse floristically homogeneous surface areas the same type (one says, the same *association*) that is the logical analog of a species.
From the particular to the general, from the concrete to the abstract, from the individual to the species, from the bunch of plants to the *association* (or ecological type), this is always for the spirit the same ascending hierarchical movement; a sort of passage to the limit causing degrees of knowledge to climb, and also, in a certain sense, those of being.

The mathematical imagination will aid us in representing this transition.

§9 Filter and Limit

The mathematical notion of *filter* which serves for the study of limits in the most general spaces was introduced in 1937 by our Master H. Cartan. But more so than to N. Bourbaki, we have recourse to a distant precursor, William of Ockham (or Occam…; named after his birthplace), the *Venerabilis Inceptor* (Venerable Enterpriser, or beginner, not doctor…), an English Friar Minor who went through Europe, taught in Paris at the start of the 14th century… (about Ockham we may refer to: “The Physics of William of Ockham”, By André Goddu, Published by Brill Archive, 1984). In the album of philosophers the name of Ockham is illustrated by a razor: one calls *Ockham's razor* the principle according to which one should not without necessity postulate beings:

Entia non sunt multiplicanda praeter necessitatem.

In his monumental history of cosmological doctrines (P. Duhem, Le Système du Monde - The System of the World -, Hermann, Vol. VII, pp. 236 sqq.), Duhem notes that:

“In accordance with this principle, Ockham uses for place the very definition of Aristotle, returning it to its fundamental simplicity, clearing it of every parasitic addition: place, this is the part of the containing body which reaches immediately the contained body.”

This is the occassion for the *Venerabilis Inceptor* to employ the razor for a fine topological dissection. From the Latin original, P. Duhem translates Ockham:

“Place is that which is ultimate in the containing, that is, the ultimate part of the containing body. It is not that there exists a certain ultimate part that is, in its totality, distinct from other parts. I name ultimate part every part which extends as far as the located body, which affects the located body in the place; according to this manner of speaking, the ultimate part has itself a multitude of parts that do not affect the located body.
But, you say, I take the ultimate part, that which, to begin with, is called place; the former does not have certain parts that affect the located body and others that do not affect it; otherwise this is not this part, but a part of this part, which would receive in the first instance the name of place.
I respond that one must distinguish on the subject of the ultimate part.
In a first way, one names ultimate part every part that extends as far as the located body and which affects immediately the contained body at the place; in this sense, there is an infinity of ultimate parts that, all, are place. If an ultimate part affects, by its right-hand side, the located body, that right half of this tangent [literally: *touching*, JPB] part is still the ultimate part, and the half of this half is also an ultimate part, and so to infinity.
In a second way, one names the ultimate part contiguous to the located body that which is found after every other, contiguous to the located body, part. In this latter sense, there is no part that is an ultimate part.”

In contemporary mathematical language a set $\mathfrak{F}$ of non-empty parts of a set **E** is called a filter (filter base) if given two of its parts, $\mathbf{F}$ and $\mathbf{F'}$:

$$\mathbf{F} \in \mathfrak{F} \quad ; \quad \mathbf{F'} \in \mathfrak{F} \quad ;$$

one can find a third one , $\mathbf{F''}$, that is contained at the same time in $\mathbf{F}$ and $\mathbf{F'}$:

$$\mathbf{F} \supset \mathbf{F''} \; ; \quad \mathbf{F'} \supset \mathbf{F''} \; ;$$

One recognizes a property that Ockham attributes to ultimate parts: however closely one restricts an ultimate part, one can restrict it more: under the name of ultimate parts, Ockham describes the filter of neighborhoods of the contained in the containing (cf. *figure 2*)

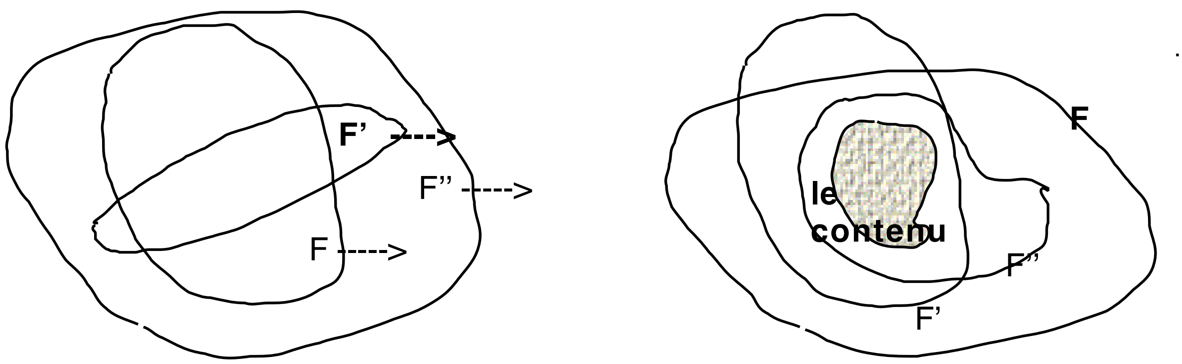


figure 2 : Two examples of filter :

Left: the place of the infinite ; Right: The filter of neighborhoods of the contained.
(at left, the infinite is outside F, F', F”...) (at Right, the shaded area is: the contained.)

Let us call now, with P.S. Alexandrov, “neighborhood of the infinite” the exterior of a bounded, but arbitrarily large, region of space: just as the location of the contained is defined in the containing by a filter, in the same way the filter of neighborhoods of the infinite defines what one will call, to follow Ockham, the place of the infinite (cf. *figure 2*).
One can further adjoin, to a space E, an ideal place: L, defined by a filter of ultimate parts: F, to a space E; the place L is not identified with any part: F; every part F has itself a multitude of parts that do not affect L; and from which one can strip out F; without, however, defining L completely purely, because “In this last sense, there is no part that is the ultimate part”.
We consider that it is similarly that one can, at a given level of the real, define as the limit place of a filter a being of higher hierarchical level.

§10 Definition by a Filter

One said that one must not be proud of appearances: it is however according to them that we know every thing. The first sight is often deceptive: but more and more attentive considerations lead us quite close to the essence of the object to deserve in the end to win our assent. In the half-light everything which rustles appears living; and silence seems the sign of solitude or of death. Let us stop: this noise was but the cracking of a branch under our step. A faint light caresses a wing that one feels to quiver, that one believes to brush past. This wing is a dry leaf that is stirred again in the air, that our arrival has shaken.
The exterior indications of life are neither necessary traits nor sufficient traits: where life is present, they can be missing; and their presence does not imply that of life. Based on more profound characteristics, a sophisticated automat will again deceive us. From qualities associated with life, only the ultimate part allows a definite definition. One therefore conceives that an ideal limit notion - to be living, to be good - is defined as comprehension by a filter on a set of the qualities.
Rather than trying to decide on formulas, let us look at an example (cf. *figure 3*); where the qualities possessed by an individual are shaded.

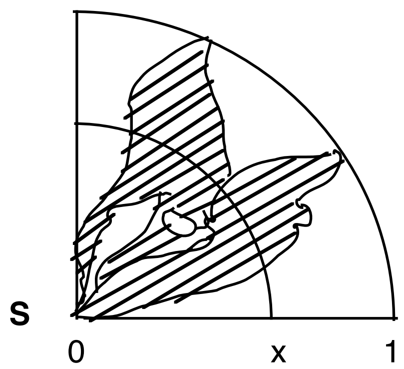


figure 3

The set Q_1 of the qualities that an individual is liable to possess is represented by the interior of a quadrant of a circle of radius 1.
(We say interior so that the vertex, S, does not represent a quality that would be perfection.) We suppose that the more a quality is distant from S, the less profound its contribution is to the limit notion **L** that we seek to define: for example, if **L** is life, the faculty of emitting a sound spontaneously will be at the distance 0.9; while the faculty of responding by a sound to a light will be at the distance 0.8.

One can postulate that an individual who possesses the set Q**x** of qualities situated a distance less than **x** from S (this distance, **x**, is a positive number less than 1; Q**x** is a quadrant of the circle of radius **x**) surely goes into the extension of the limit notion, **L**. But perhaps this is to make too few cases of qualities distant from S: even if none among them, considered in particular, is indispensable to the notion **L** (as to the notion of man, neither hearing nor sight are essential since there are deaf and blind people), should not a certain part amongst them always remain? One will say then for example that for an individual to possess the notion **L** he should possess a part at least equal to (1 - **x**) of the qualities situated at distance **x**: that is to say, 10% of those situated at the distance **x** = 0.9; half of those situated at the distance 0.5; 90% of those situated at the distance 0.1; etc.

We do not know if formal logic will come soon to treat as filters the systems of embedded conditions that define a limit notion. Our Figure 3 is certainly only a sketch which suffices however to show that logical properties of a limit notion **L** are not those of a common predicate.

We will clarify this in considering negation.

If a notion, or predicate **R**, is defined by its extension it is not on another plane than its negation (represented in §4 by **zR**). One has **I**, the set of all individuals i.
We may write, :

$$\mathbf{I}\,(\mathbf{R}) = \{\ \mathbf{i}\ \mid\ \mathbf{i} \in \mathbf{I}\ ;\ \mathbf{R}(\mathbf{i}) \text{ is true}\ \}\ ;$$
$$\mathbf{I}\,(\mathbf{zR}) = \{\ \mathbf{i}\ \mid\ \mathbf{i} \in \mathbf{I}\ ;\ \mathbf{R}(\mathbf{i}) \text{ is false}\ \}\ ;$$

each of those two sets can be defined as the complementary of the other inside **I**:

More independently of every model (that is, from a point of view which is in a certain sense that of comprehension, cf. above in §6) one sees that if in the basis vocabulary of a theory the predicate **R** appears it is possible to suppress it, and to introduce **R'**, which will have the meaning of **zR**, and will be, in its place, a basic predicate (that is to say that in the axioms of the theory, every time **R** occurs, one will replace **R** by the negation of **R'**).
But while to a part is put opposite its complement, a filter $\mathfrak{F}$ (of parts of a set **E**) does not have a complement; (the set Part(**E**) - $\mathfrak{F}$, the parts of **E** that are not in the filter $\mathfrak{F}$, is not a filter; the set $\{\mathbf{G} \mid (\mathbf{E} - \mathbf{G}) \in \mathfrak{F}\}$, of complements in **E** of parts that are of $\mathfrak{F}$ is not a filter either). It is not possible to put opposite to a limit notion **L** (defined as is suggested in Figure 3, or in any other analogous fashion) a contrary notion, **zL**, liable to be defined in the same way as **L**. In this sense a notion limit has not an opposite that is on the same plane as itself.

This holds in particular for the notion of form.
Thus "to have the form of the letter **A**" does not have an opposite: the set of patterns that are called "**A**" is not defined with respect to an ideal structure, to which an opposite structure could respond, which can define the "non-**A**". In a printed text, the "non-**A**" is defined as: "that which is **B** or **C** or **D** ... or **Z** ... or **9**"; but in general the set of patternless objects is the inevitable context of every problem of pattern recognition.

Life defines itself; while death, indeed the inanimate, is only defined relative to life.

§11 The Filter of the "secundum quid"

An aircraft flies. In the jet engines a continuous stream of kerosene is on fire; through the nozzles a jet of burning gas escapes. Sometimes along the wing or the center-board, a flap tilts, inclining with it the docile flight of the aircraft: thus this flap, that a weak enough force suffices to move, is the master of the aircraft whose mass is more than one hundred times greater. All however - flow of fuel and coordinated interplay of flaps - is governed by the light levers which obey the members of a man: this man, towards whom everything in the aircraft converges, drives the flight.

What drives the man? Can one not within the human body, as within the flying machine, climb the hierarchy of a filter?
Let us take away integument and carcass, intake of air and food, rate of oxygen and of sugar, pivots and springs; because we know like Alexandrov that what we are seeking is beyond these organs and these bounded functions.

What remains: some relays where information that flows is reverberating, coded as impulsions, as electrochemical waves, as molecules, perhaps articulated: the brain!

Without doubt; but, from the vegetative stream that we thought at first to have completely discarded, relay and information are not absolutely separated: the neurons are, on the contrary, the seat of a metabolism of the most active.

It does not suffice to carve up in the space of the body a filter of ultimate parts closing in around that which we seek: it must be filtered on site; to distinguish, within a single apparatus, levels that spatially coexist; separating that which appears lower and material so as to keep only that which formally controls it; to dissect the ultimate parts by circumscribing them, not by a volume but by a definition:

F_{n+1}, this is F_n, considered in how it is active and formal; this is F_n, restrained according to some condition, F_n *secundum quid* as would have said a Doctor of the century of Saint Louis.

In the end, from condition to condition, one realizes that everything balances on an evanescent interplay of matter and energy.

The soul is not at the end of a scalpel; it is at the end of Ockham's razor.